8Mind

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Why Eight

On the recurrence of eight in physics, mathematics, biology and human tradition, and on a test of whether eight-structured algebra helps a learning machine.

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“What I cannot create, I do not understand.”

Richard Feynman, from his blackboard at Caltech, 1988

Abstract

The integer eight appears in the valence shell of the atom, the gauge structure of the strong force, the classification of division algebras, the exceptional Lie groups, the triality of Spin(8), the periodicity of real K-theory, the protein core of chromatin and the smallest dimension that admits bound entanglement. It also appears, independently, in six religious and cultural traditions. We trace each case to its cause. The physical and mathematical cases do not share one: the octonions, triality, E8 and Bott periodicity are a single mathematical structure; the two SU(3) eights are a second; the octet rule, the histone octamer and the bound-entanglement threshold are unrelated to those and to each other. Most of them are counts attached to a symmetry group. We then propose a research program: train a language model in a closed loop against automatic verifiers, and run a controlled experiment that tests whether octonionic layers, an eight-structured prior with a matched four-dimensional control, improve a model on verifiable scientific tasks. Those

tasks carry none of the symmetries above, so the experiment tests a structured prior, not an equivariance. The null result is publishable and the experiment is designed to produce it if it is true.

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1. The pattern

Eight recurs for several unrelated reasons; this paper traces each to its cause.

We do not claim a single hidden law behind it. We claim something narrower and easier to defend: eight is one of the places where nature's structure and human pattern-seeking overlap, and that overlap is what 8Mind studies.

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2. Eight in nature

Established results in five fields turn on the number eight; each is traced below to its cause. A result is *theorem-level* when the integer is forced by a proof and cannot be moved by convention or choice of units; Section 7 uses the distinction to test for selection bias.

FIELD	THE EIGHT	WHY IT IS EIGHT
Chemistry	The octet rule: main-group atoms are most stable with eight outer electrons [1, 2]	The outer shell holds one s orbital and three p orbitals, two electrons each: 2 × (1 + 3) = 8
Particle physics	The Eightfold Way: hadrons sort into families of eight [3, 4]	The symmetry group SU(3) has $3^2 - 1 = 8$ generators
Particle physics	Eight gluons carry the strong force that binds quarks [5]	The same count, from the colour symmetry SU(3)
Mathematics	The octonions, an eight-dimensional number system [6, 7]	Hurwitz's theorem: normed division algebras exist only in dimensions 1, 2, 4 and 8. Eight is the last

Mathematics	E8, the largest exceptional Lie group, has rank 8. Its lattice gives the densest packing of spheres in eight dimensions [9]	Its Lie algebra is 248-dimensional; the eight is its rank
Mathematics	Triality: Spin(8) is the only simple Lie group with three eight-dimensional representations permuted by an outer automorphism [7, 38]	The Dynkin diagram D_4 has threefold symmetry; no other Dynkin diagram has it. Unique to dimension 8
Mathematics and physics	Bott periodicity: the stable homotopy of the orthogonal group repeats with period 8 [10, 11, 12]	Real Clifford algebras repeat with period 8; eight real symmetry classes of topological matter follow
Biology	The histone octamer: DNA in eukaryotic cells wraps around cores of eight proteins [13]	Two copies each of four histones, H2A, H2B, H3 and H4: 2 × 4 = 8
Quantum information	Bound entanglement: eight is the smallest dimension in which two systems can be entangled with no way to extract a usable pair [42, 44, 45]	The partial-transpose test is exact for 2 × 2 and 2 × 3; it first fails at 2 × 4 = 8, where undistillable states appear

2.1 The octet

The Coulomb potential of the nucleus is spherically symmetric, so single-electron states carry irreducible representations of the rotation group SO(3), of dimension $2\ell + 1$. The valence shell of a main-group atom is built from $\ell = 0$ (one s orbital) and $\ell = 1$ (three p orbitals). Spin doubles each. A closed ns^2np^6 configuration therefore holds $2 \times (1 + 3) = 8$ electrons. Lewis proposed the rule of eight in 1916 [1] and Langmuir named the octet in 1919 [2]. The rule fails where d orbitals or electron deficiency enter: hydrogen and helium, boron compounds, the transition metals and the hypervalent species.

2.2 SU(3): the Eightfold Way and the gluons

The group SU(N) has $N^2 - 1$ generators, so SU(3) has eight, conventionally the Gell-Mann matrices $\lambda_1 \dots \lambda_8$. Two distinct SU(3) symmetries occur in the strong interaction. Flavour SU(3) is approximate, broken by the quark mass differences; under it the mesons decompose as $3 \otimes \bar{3} = 8 \oplus 1$ and the baryons as $3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$, which is the octet structure Gell-Mann and Ne'eman identified [3, 4]. Colour SU(3) is exact and is the gauge symmetry of quantum chromodynamics; its gauge bosons transform in

the adjoint representation, of dimension eight [5]. There is no ninth gluon because the gauge group is SU(3) and not U(3): the colour-singlet combination is absent.

2.3 The division algebras

Hurwitz proved that a normed division algebra over the reals has dimension 1, 2, 4 or 8: the reals, complex numbers, quaternions and octonions [6]. Each Cayley–Dickson doubling gives up a property: order, then commutativity, then associativity. The next doubling, to sixteen dimensions, produces zero divisors; the Cayley–Dickson sequence continues, but no later term is a division algebra. The octonions are non-associative but alternative, and their automorphism group is the exceptional group G_2 [7]. The subgroup of G_2 that fixes one of the seven imaginary octonion units is exactly SU(3) [7], so SU(3) sits inside Aut(O) = G_2. But the octonions transform under that SU(3) as $1 \oplus 1 \oplus 3 \oplus \bar{3}$, not as the gluon octet: the theorem links the two groups, and the two eights remain different eights, the dimension of an algebra and the dimension of an adjoint representation. The same list 1, 2, 4, 8 reappears in topology: the only parallelizable spheres are S^0, S^1, S^3 and S^7, settled by Kervaire and by Bott and Milnor in 1958, with Adams's theorem on the Hopf invariant giving the stronger statement that these are the only spheres that are H-spaces [8].

2.4 E8

E8 is the largest of the five exceptional simple Lie groups: rank 8, dimension 248, with 240 roots. Its root lattice is the unique even unimodular lattice in eight dimensions. Viazovska proved in 2016 (published 2017) that this lattice gives the densest sphere packing in eight dimensions, with density $\pi^4/384 \approx 0.2537$ [9]. The octonions and E8 are linked: the integral octonions form a copy of the E8 lattice [7]. The same method settled dimension 24, through the Leech lattice.

2.5 Triality

The rotation group in n dimensions, SO(n), always has an n-dimensional vector representation and, through its double cover Spin(n), spinor representations; for even n there are two half-spinor representations, each of dimension $2^{n/2-1}$. Vector and spinor dimensions coincide only at n = 8: the vector, the spinor and the conjugate spinor of Spin(8) are all eight-dimensional, and there is an outer automorphism of the group that permutes the three. This is triality, named by Cartan in 1925 [38]. Triality and the octonions are equivalent structures: each can be constructed from the other, and the exceptional groups F_4, E_6, E_7 and E_8 are all built from the octonions by the Freudenthal–Tits

construction [7]. In string theory the eight transverse dimensions of the superstring use the same coincidence to match bosons to fermions [41]. The rank of E8 is a property of one group; triality is a property of dimension eight itself.

2.6 Bott periodicity

Bott showed that the stable homotopy groups of the orthogonal group repeat with period eight: $\pi_{k+8}(O) = \pi_k(O)$, with the sequence $\mathbb{Z}_2$, $\mathbb{Z}_2$, 0, $\mathbb{Z}$, 0, 0, 0, $\mathbb{Z}$ for k = 0 ... 7 [10]. The algebraic counterpart is the periodicity of real Clifford algebras, $Cl_{n+8} \cong Cl_n \otimes M_{16}(\mathbb{R})$ [11]. This period has a direct physical consequence. In the classification of topological insulators and superconductors, the eight real symmetry classes repeat with period eight in spatial dimension [12].

2.7 The histone octamer

The nucleosome core particle wraps about 146 base pairs of DNA in 1.65 left-handed superhelical turns around a protein core [13]. The core is an $(H3–H4)_2$ tetramer flanked by two H2A–H2B dimers, related by a twofold (dyad) axis. The eight here is 2 × 4: a discrete twofold symmetry acting on four histone types. It is conserved across eukaryotes, but it is a fact of molecular evolution, not a consequence of a conservation law.

2.8 Where distillation stops: eight in quantum information

One further eight is a threshold rather than a count. Take two quantum systems of dimensions m and n. In 1996 Peres proposed a test for whether their joint state is entangled: transpose one half of the density matrix and check whether the result is still a valid state [43]. The same year the Horodeckis proved that this test is exact, both necessary and sufficient, precisely when $m \times n \leq 6$, that is for 2 × 2 and 2 × 3 [42]. The first dimensions where it fails are 2 × 4 = 8 and 3 × 3 = 9. In 1997 Paweł Horodecki built explicit states in those dimensions that pass the test yet are entangled [44], and in 1998 the three Horodeckis proved that no such state can be distilled [45]: no sequence of local operations and classical communication, applied to any number of copies, yields a single maximally entangled pair. Such entanglement is called *bound*. Eight is the smallest total dimension at which it occurs. Below it, every entangled state can be distilled; at it, some cannot.

This eight is theorem-level, and it differs in kind from the other rows: most of those are counts attached to a group; this one is a threshold, and its coincidence with them is

numerical, since 2 × 4 as a Hilbert-space dimension has no relation to any Lie-algebra eight.[2]

The octet rule is a rule of thumb with known exceptions, and nature builds with other numbers too: three quarks in a proton, four bases in DNA, twenty amino acids.

Eight arrives by five independent routes: the atom's rotational symmetry; the group SU(3), whose two eights are one count; the histone core; one structure in mathematics that appears as the octonions, triality, E8 and Bott periodicity, the last generated by the octonionic line bundle over OP^1 [7]; and the threshold of Section 2.8. The embedding $SU(3) \subset G_2$ of Section 2.3 relates the groups, not the counts.

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3. Eight and symmetry

The routes are separate, but most of the rows pass through one idea: symmetry. Two of them do not even share arithmetic: the octet is 2 × (1 + 3) and the SU(3) eight is $3^2 - 1$. A system has a symmetry when something about it can be changed and nothing observable differs.

In 1918 Emmy Noether proved that every continuous symmetry of physical law yields a conserved quantity [14].

Statement. Let a system have Lagrangian $L(q, \dot{q}, t)$, with action $S = \int L\, dt$ stationary on the true path [34]. If the Lagrangian is invariant under the infinitesimal transformation $q_i \to q_i + \varepsilon\, K_i(q)$ (invariance up to a total time derivative adds one term to Q), then on solutions of the equations of motion the quantity

$$Q = \Sigma_i \, (\partial L / \partial \dot{q}_i) \, K_i$$

is constant in time. For $t \to t + \varepsilon$ the same argument gives the energy, $\Sigma_i \, p_i \, \dot{q}_i - L$. A symmetry group with n independent generators gives n conserved quantities.

SYMMETRY	GENERATORS	CONSERVED QUANTITY
Translation in time	1	Energy
Translation in space	3	Momentum
Rotation, SO(3)	3	Angular momentum

Phase, U(1)	1	Electric charge
Flavour, SU(3)	8	Eight flavour charges, approximately conserved

The last row is the one to read with care. The eight gluons are the gauge fields of colour SU(3), a local symmetry, and for a local symmetry Noether's first theorem does not deliver observable conserved charges: colour charge is confined and the colour current is gauge-variant. The clean Noether example is the global flavour SU(3), whose eight charges are approximately conserved, broken by the quark mass differences, which is why the Eightfold Way works as a classification and fails as an exact law.

The histone octamer and the octonions reach eight by other routes, and symmetry yields many other numbers: SU(2) has three generators, SU(5) has twenty-four. It does not favour eight.

Where the theorem stops. Noether's theorem applies to continuous symmetries of an action. Discrete symmetries, such as the dyad axis of the nucleosome, give no conserved current. And where a symmetry is absent the conservation law fails: in an expanding universe the background is not invariant under time translation, and total energy is not conserved in the ordinary sense. A research program built on symmetry has to locate the breaks as carefully as the symmetries.

3.1 The octet, drawn

Two of the eight generators of SU(3), T_3 and T_8, commute; they label every state by isospin I_3 and hypercharge Y, the axes of a plane. The other six shift a state one step along one of six directions in that plane. Figure 1 shows the result: six states on the vertices and two at the centre, 6 + 2 = 8.

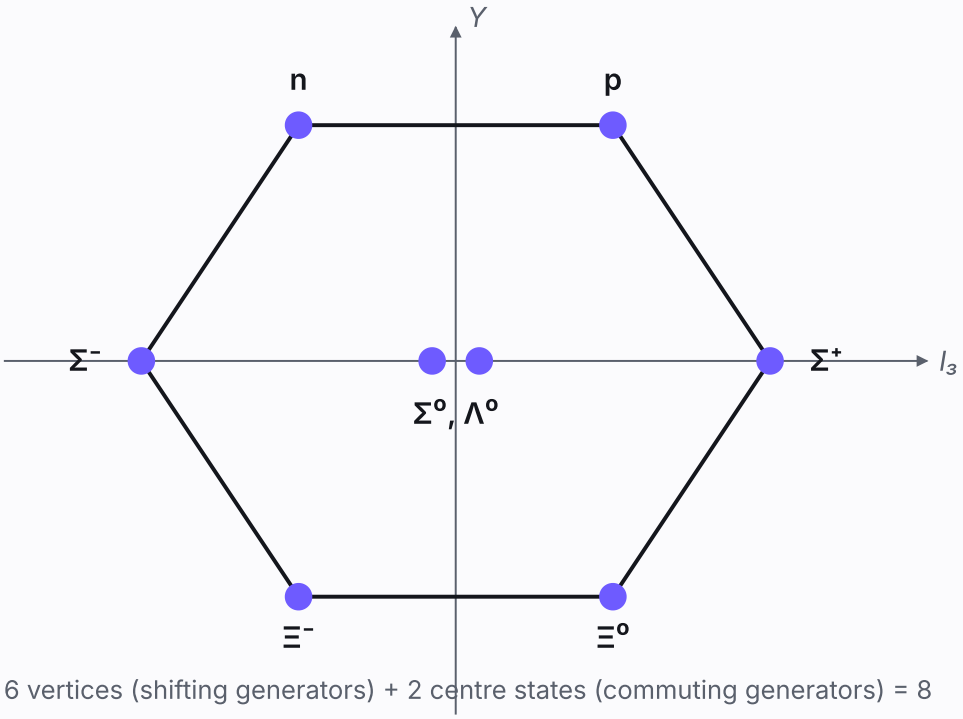


Figure 1. The spin-½ baryon octet on the (I_3, Y) plane. The meson octet and the gluon octet have the same shape with different labels. After Gell-Mann and Ne'eman [3, 4].

3.2 The division algebras and the Standard Model

The division-algebra sequence 1, 2, 4, 8 and the gauge groups of the Standard Model line up suggestively: the complex numbers carry U(1), the quaternions carry SU(2), and SU(3) sits inside the automorphisms of the octonions (Section 2.3). A line of work going back to Günaydin and Gürsey in 1973 has tried to derive the Standard Model's gauge structure from this sequence [35, 36, 37]. It is a research program with open problems, not an accepted result; this paper does not rely on it.

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4. Claims and the research question

Two claims.

1. Eight is forced by theorems in particle physics, mathematics and quantum information, and is an established empirical fact in chemistry and cell biology. Whether it is over-represented among small integers is open; Section 7 states the test.
2. Eight is culturally real: independent traditions built meaning on it by unrelated routes (Appendix A).

We do not claim that the eight of physics causes the eight of tradition, or that eight dimensions describe physical reality. This thesis does not depend on either.

> **What 8Mind builds.** Science models trained against verifiers. The founding hypothesis, tested in Section 6, is that the algebra behind several of the eights of Section 2 can be built into such a model to its advantage; symmetry gives the question a direction, because an architecture can be built to respect one.

Formally. A map f between representation spaces of a group G is equivariant if $f(\rho_{in}(g)\, x) = \rho_{out}(g)\, f(x)$ for all g in G. Building equivariance into a network restricts its hypothesis space to functions consistent with the symmetry, which reduces the data needed to learn them [26, 27]. This is established practice: equivariant architectures underlie protein-structure prediction [28] and interatomic potentials [29]. Whether built-in structure beats learned structure at fixed compute is an empirical question, and Arm A against Arm B in Section 6 is that question.

What the experiment can and cannot test. Equivariance helps when the data carry the symmetry. The primary task of Section 6, formal proof search in Lean, carries none of the symmetries of Section 2: no SO(3), SU(3), G_2 or Spin(8) acts on proof text. Arms B to D therefore test a structured weight-sharing prior, octonionic against quaternionic against unstructured, and not an equivariance. A null result on them would say nothing about equivariant models on data that have the symmetry, which already exist and already work [28, 29]. The symmetry rationale is tested only on the secondary tasks with known symmetry content.

Prior work on eight-structured models. The components already exist and the paper does not claim them. Octonion-valued networks have been trained on image tasks [30]. The E8 lattice is used as a codebook for compressing large language models [31]. What has not been done, to our knowledge, is a controlled comparison of these structures on verifier-scored scientific reasoning, motivated by the cross-domain recurrence set out above.

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5. The method: the loop

The method behind the fastest recent progress in AI is a loop: the model proposes, a verifier scores, the model learns from the score, and the cycle repeats. It works where a machine can check the answer.

In the notation of reinforcement learning: a policy π_θ produces a candidate y for a problem x; a verifier V returns a score V(x, y); training maximizes the expected score E[V(x, y)] over θ. The verifier replaces human labels. Its soundness bounds what the loop can learn.

SYSTEM	THE LOOP	RESULT
AlphaZero [21]	Self-play, with the outcome of the game as referee	Superhuman chess, shogi and Go from the rules alone
AlphaProof [22]	Reinforcement learning inside the Lean proof checker, on about 80 million auto-formalized statements	With AlphaGeometry 2, silver-medal standard at the 2024 International Mathematical Olympiad
DeepSeek-R1 [23]	Reinforcement learning on math and code problems, rewarded only for verifiably correct answers	Reasoning ability emerged without human-written reasoning examples (R1-Zero)
AlphaEvolve [24]	A language model proposes programs, automated evaluators score them, the best are evolved further	4 × 4 matrix multiplication over the complex numbers in 48 multiplications, against 49 from two levels of Strassen's 1969 method [33] (AlphaTensor had reached 47 in characteristic 2 in 2022)
AI co-scientist [25]	Agents generate, debate and evolve hypotheses; the laboratory is the final check	Drug-repurposing and target-discovery leads validated in vitro and in organoids

The constraint. The loop is only as good as its verifier. Mathematics and code went first because a proof checker and a test suite already existed; AlphaEvolve's authors name the same limit [24].

The cost of a verifier. A Lean check is exact and takes seconds. A density-functional calculation is an approximation with known failure modes and takes minutes, so the chemistry loop is slower and its reward noisier. The program takes mathematics first for that reason and treats the chemistry verifier as a research object: the reward carries an uncertainty estimated from an ensemble of functionals, and candidates scoring within that uncertainty are re-scored at a higher level of theory before any reward is paid.

The order of attack. Most of chemistry, materials science, biology and physics has no cheap, trusted, automatic check; 8Mind's first work is those verifiers. It starts where a

verifier exists or can be built from simulation, then moves outward.

1. Mathematics: formal proof checking in Lean.
2. Chemistry and materials: quantum-chemistry and molecular simulation as the scorer.
3. Biology: structure prediction as the scorer, with laboratory partners for confirmation.
4. Theoretical physics: the hardest case and the long-term goal, approached through its mathematics.

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6. The research program

8Mind proposes an 18-month program: build a proprietary science model on an open foundation, train it with the loop in two verifier environments, and run one controlled test of eight-structured mathematics.

1. **Foundation (months 0 to 3).** Start from an open-weight model of a few billion parameters, a scale at which reinforcement learning on verifiable rewards has been shown to work.
2. **Domain training (months 3 to 9).** Continue training on a private corpus drawn from mathematics, chemistry, physics and biology; the texts of the traditions in Appendix A are not training data. Build the first two verifier environments: formal mathematics and computational chemistry.
3. **The loop (months 6 to 15).** Train with reinforcement learning on verifiable rewards inside those environments. At run time the model works as an agent: it calls the prover or the simulator, reads the result and tries again.
4. **The eight experiment (months 9 to 15).** Train models that differ in one respect only, octonionic layers against none, with a quaternionic arm to separate eight from structure in general (Section 6.1). Publish the comparison whichever way it falls.

6.1 Design of the eight experiment

ARM	WHAT CHANGES	RATIONALE
A. Baseline	Standard transformer, real-valued weights	Control

B. Octonionic	Selected linear maps constrained to octonion multiplication, which shares weights across eight real components	Tests the last division algebra; a constrained map has about one-eighth the free parameters, so Arm B is widened to match Arm A's count [30]
C. E8 lattice (robustness control)	Embeddings or weight blocks quantized to E8 lattice points	A lossy compression, not a symmetry prior: tests whether the model tolerates the densest eight-dimensional code without loss [9, 31]
D. Matched non-eight	The same constructions in dimension 4 (quaternions, D4 lattice)	Separates an effect of eight from an effect of structure in general

Controls. Equal parameter count and equal compute across arms; at least five random seeds per arm; primary endpoint, test set and analysis plan registered before training. Primary endpoint: verifier pass rate on held-out formal-mathematics problems, miniF2F [39], for one pre-registered comparison, with the minimum effect of interest and the seed count fixed in the pre-registration. Secondary: PutnamBench [40], sample efficiency, and performance on tasks with known symmetry content, corrected for multiplicity. Results reported as effect sizes with confidence intervals.

A known technical obstacle. Octonion multiplication is non-associative, so stacked octonionic layers do not compose to a single octonionic map, and any product of weights depends on its bracketing. Arm B will fix the bracketing by convention and report sensitivity to the choice. Equivariance under G_2, the automorphism group of the octonions, is the principled alternative and is a candidate for a follow-up arm.

Limits of the design. Widening Arm B to match parameters changes its depth-to-width ratio; Arm C is lossy by construction; reinforcement learning at few-billion-parameter scale varies across seeds. Each is recorded as a confound in the pre-registration, and the seed count is set with the last in view.

Either outcome has value. If an eight-structured model beats the baseline on the primary endpoint, that is a new finding. If it does not, 8Mind still holds working verifiers for two sciences, which is the asset the loop depends on.

7. Limits and objections

Selection bias. Small integers recur throughout mathematics because there are few of them and many patterns [32]. A list of eights can be matched by a list of sevens or twelves. The paper's defence is that it does not argue from the count of examples: it traces each case to its cause, and finds a shared mechanism (symmetry) in a subset.

A test of the objection. The same survey, applied to 6, 7, 12 and 24 with the theorem-level criterion of Section 2 and the same search of the same sources, would show whether eight is over-represented among theorem-level results (Hurwitz, Bott, Viazovska) as against counts by convention (the number of planets, the days of a week). It is part of the first phase of work and its result will be reported.

The gap between sections. Sections 2 and 3 and Appendix A establish that eight is real in nature and in tradition. They do not establish that eight-structured algebra helps a learning machine. Section 6 is the experiment that addresses that gap, and Arm D exists so that a positive result cannot be attributed to eight without evidence.

Tradition as data. The traditions enter the paper as a record of human pattern-finding, and the program only as a held-out probe set, not as training data. No theological claim is treated as evidence about physics.

Appendix A. Eight in human tradition

Six traditions, on three continents and across roughly 5,000 years, give eight a privileged place; China does so twice. The routes differ.

TRADITION	THE EIGHT	ROUTE TO EIGHT
Mesopotamia	The eight-pointed star of Inanna and Ishtar, on seals and reliefs from the third millennium BCE [15]	The sky: by the Old Babylonian period the star stood for the planet Venus
Egypt	The Ogdoad: eight primordial deities of Hermopolis, whose Egyptian name Khemenu means Eight-Town [16]	Four qualities of the pre-creation deep (water, darkness, endlessness, hiddenness), each as a male and female pair: 4 × 2

China, I Ching	The bagua: eight trigrams describing the states of reality [17]	Three lines, each broken or unbroken: 2 × 2 × 2
China, language	Eight as the luckiest number	Sound: eight is "ba", close to "fa", prosperity
Buddhism	The Eightfold Path, set out in the Buddha's first sermon; the eight-spoked dharma wheel [18]	Doctrine: eight elements of practice
Hinduism	The Ashtadikpalas, eight guardians of the directions; Ashtalakshmi, eight forms of the goddess of wealth	Space: four cardinal directions plus the four between
Judaism and Christianity	Circumcision on the eighth day; the resurrection as the "eighth day"; octagonal fonts and baptisteries [19]	Time: seven days of creation plus one, a new beginning

No common ancestor explains this table. A Chinese homophone, a Mesopotamian planet and a Hebrew week have nothing to do with each other. Two of the routes are combinatorial: the bagua is the set of all binary strings of length three, $2^3 = 8$, and the Ogdoad is 4 × 2, the same factorization as the histone octamer. The resemblance is arithmetical, not causal.

Venus. In one case the eight of nature and the eight of tradition may touch. Venus returns to the same position against the stars about every eight years, after five synodic periods: 5 × 583.92 = 2,919.6 days against 2,922.0 for eight years, and thirteen sidereal orbits, 13 × 224.70 = 2,921.1 days, a 13:8 near-commensurability to within about 2.4 days that Babylonian astronomers used to predict the planet's appearances [20]. It is not known to be a dynamical resonance, and that Ishtar's eight-pointed star records the cycle is a hypothesis, not an attested fact.

Notes

[1] Enzo Cavaliere is the founder of 8Mind and Quantum8 and the author of this document. San Francisco · Toronto.

[2] In October 2026 Wissner-Gross announced Chain Theory, a program that rebuilds physics from one axiom, that nothing exists but finitely many maximally entangled pairs of finite-dimensional systems known only by distilling them, and that takes the Horodecki

threshold as a founding clue: link types with dimension product at most six, (2,2) and (3,2), are identified with spacetime and electromagnetism; the (4,2) link, dimension eight, with the weak interaction; the strong interaction sits at (3,3), dimension nine [46]. It is days old and unreviewed, and this paper does not depend on it.

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